Abstract

Studying non-life insurance markets with Nash equilibria and dependent risk models

In non-life actuarial mathematics, different quantitative aspects of insurance activity are studied This thesis aims at explaining interactions among economic agents, namely the insured, the insurer and the market, under different perspectives. Chapter 1 emphasizes how essential the market premium is in the customer decision to lapse or to renew with the same insurer. The relevance of a market model is established.

In chapter 2, we address this issue by using noncooperative game theory to model competition. In the current literature, most competition models are reduced to an optimisation of premium volume based on the simplistic picture of an insurer against the market. Starting with a one-period model, a game of insurers is formulated, where the existence and uniqueness of a Nash equilibrium are verified. The properties of premium equilibria are examined to better understand the key factors of leadership positions over other insurers. Then, the derivation of a dynamic framework from the one-period game is done by repeating of the one-shot game over several periods. A Monte-Carlo approach is used to assess the probability of being insolvent, staying a leader, or disappearing of the insurance game. This gives further insights on the presence of non-life insurance market cycles.

A survey of computational methods of a Nash equilibrium under constraints is conducted in Chapter 3. Such generalized Nash equilibrium of n players is carried out by solving a semismooth equation based on a Karush-Kuhn-Tucker reformulation of the generalized Nash equilibrium problem. Solving semismooth equations requires using the generalized Jacobian for locally Lipschitzian function. Convergence study and method comparison are carried out.

Finally, in Chapter 4, we focus on ruin probability computation, another fundemantal point of non-life insurance. In this chapter, a risk model with dependence among claim severity or claim waiting times is studied. Asymptotics of infinite-time ruin probabilities are obtained in a wide class of risk models with dependence among claims. Furthermore, we obtain new explicit formulas for ruin probability in discrete-time. In this discrete-time framework, dependence structure analysis allows us to quantify the maximal distance between joint distribution functions of claim severity between the continuous-time and the discrete-time versions.

Keywords: Customer behavior, market cycles, ruin theory, non-life insurance, game theory, generalized Nash equilibrium computation, dependent claim severity models